Quant Puzzles Week 1 Solutions

Topics

- Capital Asset Pricing Model:

https://www.investopedia.com/terms/c/capm.asp

- Monte Carlo Simulation:

What Can The Monte Carlo Simulation Do For Your Portfolio?

Analysts can assess possible portfolio returns in many ways. The historical approach, which is the most popular, considers all the possibilities that have already happened. However, investors shouldn't stop at this. The Monte Carlo method is a stochastic (random sampling of inputs) method to solve a statistical problem, and

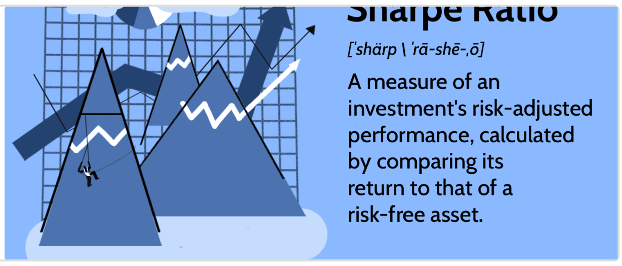
https://www.investopedia.com/articles/investing/112514/monte-carlo-simulation-basics.asp

- Sharpe Ratio

Sharpe Ratio Formula and Definition With Examples

The Sharpe ratio compares the return of an investment with its risk. It's a mathematical expression of the insight that excess returns over a period of time may signify more volatility and risk, rather than investing skill. Economist William F.

https://www.investopedia.com/terms/s/sharperatio.asp



- Marginal Sharpe Ratio

https://hal.science/hal-03189299v2/file/Computation_Marginal_Contribution_Sharpe_ratio.pdf

Question 1: Time To Positive Returns

An investment strategy has an annualized expected return of 3% and an annualized volatility of 10%. Assume returns are normally distributed. After how many years will the mean returns between years be positive with at least 91.3% probability? What about the compounded returns?

[You are allowed to use code to solve this problem if necessary]

We can simulate n runs of the returns distribution for some number of years. We can then measure what percentage of those runs are positive after a certain amount of years. Studying the results of our simulation we can then determine after how many years we would have a 91.3% probability of having positive returns. (Code is in python)

```
import random
#Computes a moving average from a gaussian sample over n trials
def gauss_moving_average(mu,sigma,n):
    moving_average_list = []
    for i in range(n):
        if i == 0:
            moving_average_list.append(random.gauss(mu,sigma))
        else:
            moving_average_list.append((moving_average_list[i-1]*i+random.gauss(mu,sigma))/(i+1))
    return moving_average_list

#Outputs a list of guassian moving average time series
def run_sims(mu, sigma, n, years):
    sims = []
    for i in range(n):
        sims.append(gauss_moving_average(mu,sigma,years))
    return sims
#Outputs the percentage of positive entries in a column of a matrix
def percent_positive_column(matrix,column):
    n = len(matrix)
    percent_positive = 0
    for i in range(n):
        if matrix[i][column] > 0:
            percent_positive+=(1/n)
    return percent_positive*100
```

```
#Runs the simulation and checks for our confidence level
matrix = run_sims(3,10,100000, 100)
for i in range(100):
    conf = percent_positive_column(matrix, i)
    print(conf)
    if  conf >= 91.3:
        print("Year",i+1, "has positive returns with probability", conf)
        break
```

This code computes it for mean returns? Can you modify the code for compounded?

Question 2

Explain what alpha and beta are. Suppose for some time period, the returns of a benchmark investment vehicle is x. A stock has a beta of 1.5, the returns of the stock are r.

$$r \leq 1.5x$$

Stock B has returns strictly equal to 1.5x. Your portfolio manager requires that you must go long one stock and short the other. Which stock do you go long and which stock do you go short?

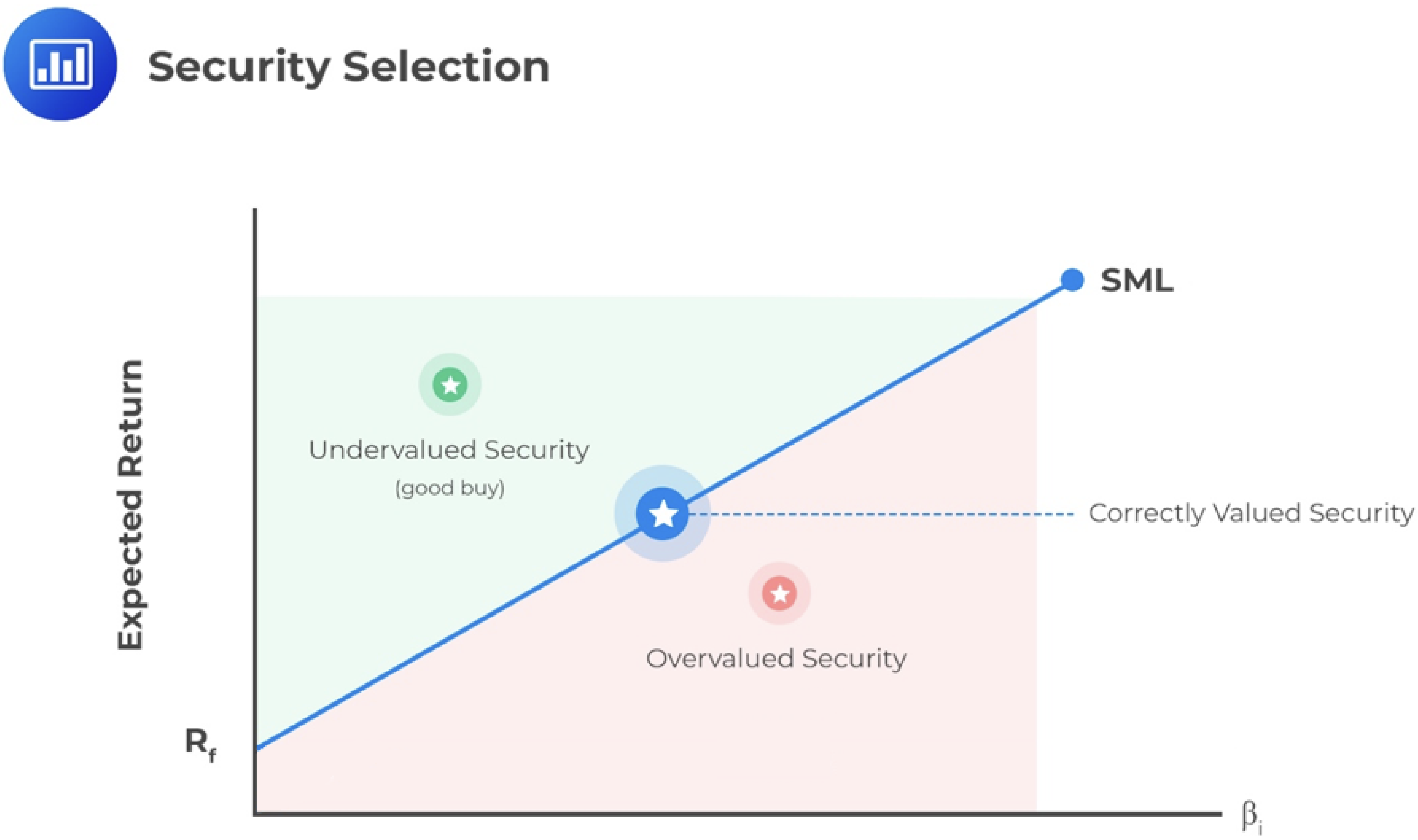
The capital asset pricing model dictates

$$R_p = (R_m - R_f)\beta + (R_f + \alpha)$$

where R_p is the expected returns of our portfolio, R_f is the risk free rate of return, β is a slope representing a linear regression coefficient, and $\alpha = I - R_f$ where I is the intercept of our linear regression.

If we take the returns of our benchmark, we can construct a line that represents our benchmark expected return for a unit β of exposure to that benchmark. (Conceptually we can increase beta by taking on leverage and paying the risk free rate. This is why it is deducted from our market returns. Beta is just a slope in relation to the excess returns of the market. This equation gives us a line. Stocks that have a given beta and fall above this line are undervalued (they deliver higher returns than what we would expect given their beta). Stocks below the line are over-valued - they deliver less returns than what we would expect given their beta.

$$y = (R_m - R_f)\beta + R_f$$



Since you have to go long one stock and short the other, you should go long the stock that is equal to 1.5x as it is fairly valued. You should short the over-valued stock.

Question 3

The sharpe ratio of our portfolio is s1, the sharpe ratio of some stock is s2. The correlation of your portfolio and the stock is p.

What is the maximum correlation that the portfolio can have to the stock such before adding the stock to our portfolio would have a negative impact on our sharpe ratio?

[Reference the other attached pdf in the solution manual, it includes a full research paper on the concept of “marginal sharpe”] The concept you need for this problem is given below:

Proposition 3. *It is optimal to include an asset i in a portfolio if and only if*

$$S_i \geq \rho_{i,p} S_p$$

Rearrange the equation to

$$S_i / S_p \geq p_{i,p}$$

Therefore, the maximum correlation before the new asset has a negative impact on our portfolio is

$$S_i / S_p$$