Quant Puzzles Week 2: Picking Bets

Question 1: One Bet

You have the option to pick one of two bets

Bet 1

- You have a 1% chance of winning $1,000,000 and a 99% chance of losing 1000

Bet 2

- You have a 85% chance of winning 1000 and a 15% chance of losing $100

Which one do you choose?

Solution 1

In interviews, you are sometimes asked questions that are debatable decision oriented questions. You will be asked to justify your reasoning. Here is reasoning for choosing bet 2.

Sharpe Ratio

The Sharpe ratio is the ratio of the expected returns to the volatility (standard deviation).

$$S = \frac{E[R]}{\sigma}$$

Sharpe Ratio of First Bet

We risk 1000 for a 1% chance of winning 1000000 and a 99% chance of losing all our money. The expected returns and the variance is given below. Recall that the variance formula

$$Var(X) = E[X^2] - E[X]^2$$

$$E[R_1] = 0.01 \cdot \frac{1000000 - 1000}{1000} + 0.99 \cdot \frac{0 - 1000}{1000} = 9$$

$$\sigma_1^2 = E[R_1^2] - E[R_1]^2 = 9900 \implies \sigma_1 \approx 99.499$$

$$S_1 = \frac{E[R_1]}{\sigma_1} \approx 0.09$$

Sharpe Ratio of Second Bet

$$E[R_2] = .85 \cdot \frac{1000 - 100}{100} + .15(\frac{-100}{100}) = 7.5$$

$$\sigma_2^2 = E[R_2^2] - E[R_2]^2 = 12.75 \implies \sigma_2 = 3.57$$

$$S_2 = \frac{E[R_2]}{\sigma_2} = 2.1$$

From a risk-adjusted returns standpoint, the second bet outperforms the first bet **by a long shot**. If you had access to some kind of leverage, you could leverage the second bet to generate the same returns as the first bet while risking considerably less.

Question 2: Building A Portfolio

Recall each of the two bets from question 1:

Bet 1

- You have a 1% chance of winning $1,000,000 and a 99% chance of losing 1000

Bet 2

- You have a 85% chance of winning 1000 and a 15% chance of losing $100

You are now allowed to build a portfolio of recurring bets. You are allowed to hold some percentage of your portfolio in cash, some percentage in bet-1 and some percentage in bet-2. What are the optimal weights to maximize the long-term growth and minimize the risk of ruin (chance that your portfolio goes down to 0)?

- Note: We are not necessarily trying to maximize the Sharpe ratio here.

Solution

Use the kelly criterion to build your portfolio

q: Probability of losing bet

p: Probability of winning bet

b: (upside)/(downside)

The optimal allocation to maximize long-term growth of a portfolio is:

$$f = \frac{bp - q}{b}$$

Bet 1

$$f = \frac{1000 \cdot 0.01 - 0.99}{1000} = 0.00901$$

Bet 2

$$f = \frac{10 \cdot 0.85 - 0.15}{10} = 0.835$$

Cash:

We hold the rest in cash

$$1 - .835 - .001 = 0.15599$$

Question 3: Measuring Performance

What is the Sharpe ratio of the portfolio you constructed in question 2?

Solution

We can construct the sharpe ratio of all three by first calculating the weighted expectation. Since the bets are all independent, we can sum up the weighted variances and take the square root for the portfolio volatility.

$$\mu = w_1 E[R_1] + w_2 E[R_2] + w_3 E[R_3]$$

$$\sigma = \sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + w_3^2\sigma_3^3}$$

from our work above, we have the weights. We can just plug this into the formula to get the Sharpe.

Remember that holding cash gives us 0 returns and has 0 volatility.

$$E[R_3] = 0, \sigma_3 = 0$$

$$S_p = \frac{w_1 E[R_1] + w_2 E[R_2]}{\sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2}} \approx 2.03$$