Quant Puzzles 9 - Multinomials

Question 1: Exactly 6 [Medium]

If I roll a die 36 times, what is the probability that each number appears exactly 6 times?

Solution:

We can use the multinomial theorem which states that for an experiment of n trials, with k possible results and $x_1 ... x_k$ desired outcomes corresponding to each of the possible results the number of possible experimental outcomes is given by the following formula:

$$\binom{n}{x_1, .., x_k} = \frac{n!}{x_1 \cdot .. \cdot x_k}$$

We know that the probability of any face from the dice showing up is $1/6$ and that we are rolling the dice 36 times. Therefore, we have the solution given below.

$$\binom{36}{6,6,6,6,6,6}(1/6)^{36} = \frac{36!}{6!^6}(\frac{1}{6})^{36}$$

Question 2: Path Probability [Medium]

You are randomly traveling through a 3-dimensional grid-space. You start at the point **(0,0,0)**, and wish to reach the point (r, k, m). At each step, you randomly pick one of the three dimensions and move one space **forward**. What is the probability that you reach point **(r,k,m)** via the following path:

- the first r steps take you to (r,0,0)
- the next k steps take you to (r,k,0)

- the last m steps take you to (r,k,m)

Solution:

This is another application of the multinomial formula. The sequence listed above gives the instructions for one of the paths so we need only calculate the total number of paths to retrieve the probability.

$$\frac{1}{\binom{r+k+m}{r,k,m}} = \frac{r! \cdot k! \cdot m!}{(r+k+m)!}$$